INTERNATIONAL DESIGN CONFERENCE - DESIGN 2004
Dubrovnik, May 18 - 21, 2004.

DESIGN 2004

ELASTIC DEFORMATIONS IN COMPUTER AIDED TOLERANCE ANALYSIS: A FOCUS ON TWO-DIMENSIONAL CONTACT SURFACES

R. Lustig, R. Hochmuth and H. Meerkamm

Keywords: tolerance analysis, elastic deformations, contact surfaces, design system mfk

1. Introduction

Some years ago the analysis and simulation of complex tolerance chains was not possible to be performed because of a lack of computer support. Nowadays modern computer based systems for calculating tolerance chains or multi-body-problems or elastic deformations are available on the market. With these systems the designer is able to model a more or less detailed view on a parts or products behaviour.

The problem on this aspect is the exclusive focus on one or another aspect. These systems do not model the effect of the interdependencies of different phenomena, such as tolerance zones, or elastic deformations. But both effects are quantitative comparable and effect on the function of a technical system with interdependencies on each other.

In this work the authors will describe the theory coupling both mentioned effects, show an application field and focus on the expansion of the theory to support kinematical effects as well as. This leads to an intelligent, function-oriented evaluation of tolerance zones and elastic deformations. The design system *mfk* with its product model therefore is the base for this mentioned analysis tools.

2. Motivation

The need of computer aided simulation tools is known in engineering design for a couple of years now. The simulation of more-dimensional tolerance chains under form-, positional- and dimensional tolerances is nowadays possible with systems as VisVSA™ or CETOL6Sigma™. The calculation of elastic product behaviour is known for longer times now. The logical close-up to the elastic behaviour of a tolerance technical system is the logical consequence. Other works are dealing with this aim, too.

Application fields for this new method are various. One can find several aspects in medicine technology, mechanical engineering and others. In all these branches the functional behaviour of a system can not be modelled without focus on both mentioned aspects.

Taking a simplified example from mechanical engineering for producing cutting-pieces needs to have an acceptable concentricity of the work piece to tooling support (figure 1). Here it is obvious that besides tolerance based deviations also elastic deformations resulting from initial tension as well as operating forces influence this quality parameter concentricity. Furthermore the kinematical position of the slider relative to the guidelines plays an important role for the resulting work piece quality. This leads further on to the kinematical aspect.

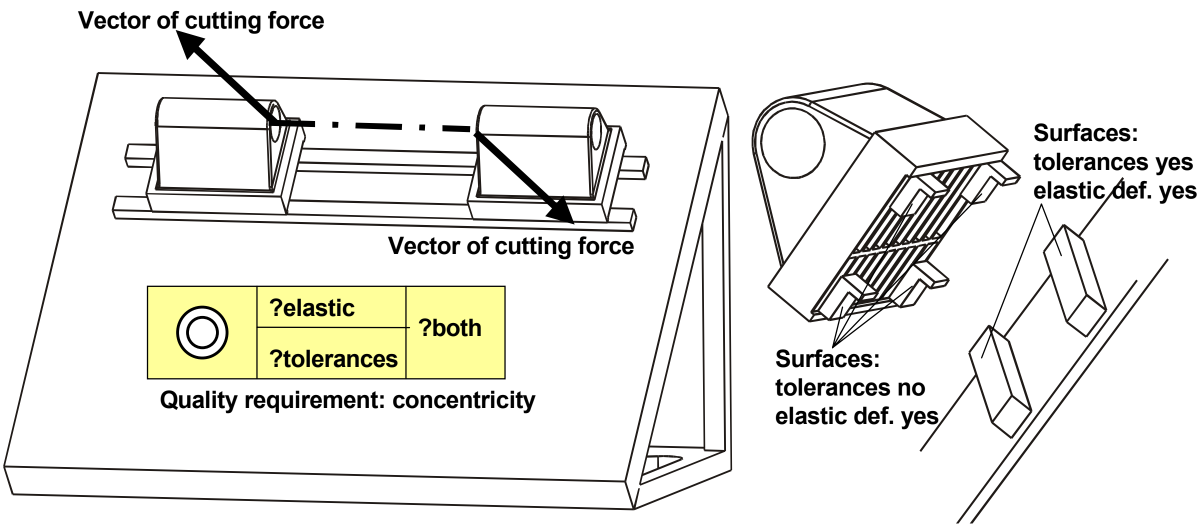


Figure 1. Motivation for the Method

3. Concept

The concept for the integration of elastic deformations and tolerance zones into computer supported analysis tools is part of the design system *mfk*. The method is based on a mathematical representation of technical surfaces (figure 2). In general every technical relevant surface can be approximated and represented by a mathematical equation. In our case of elastic deformations and tolerance zones a component of waviness is part of this equation. This results from characteristics of technical applications: e.g. regular distribution of fixing holes on flanges, n-bow equal thick in the processing of tolerances provoked by vibrations or restraints.

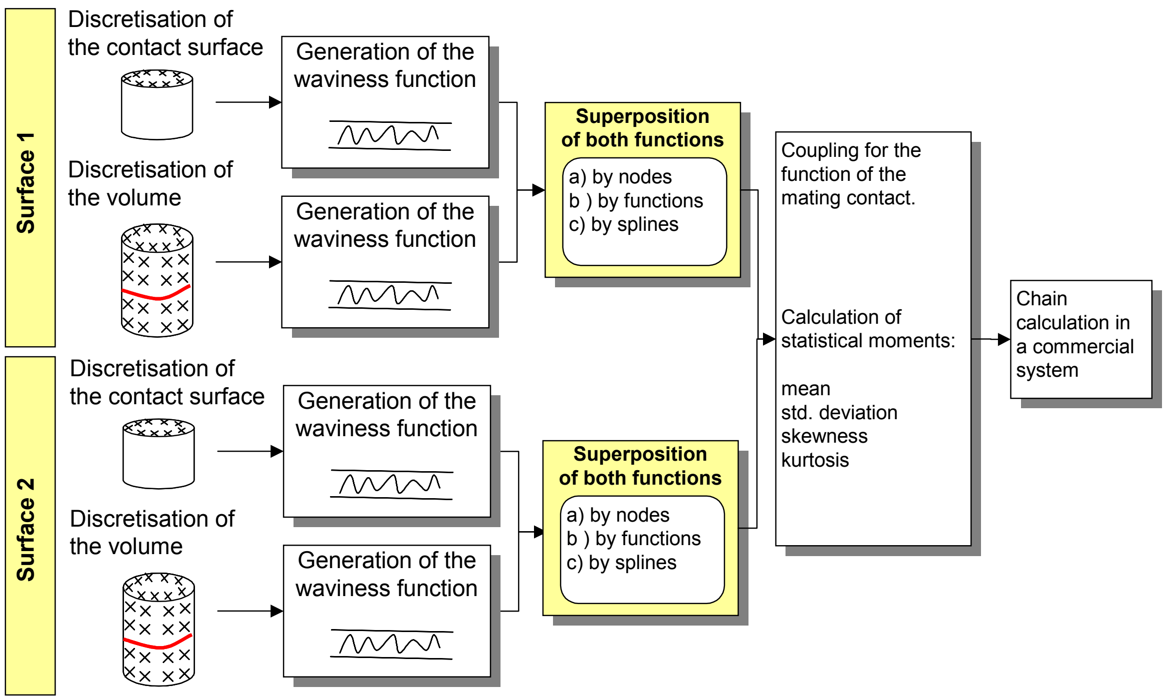


Figure 2. Basic Concept of the linking procedure

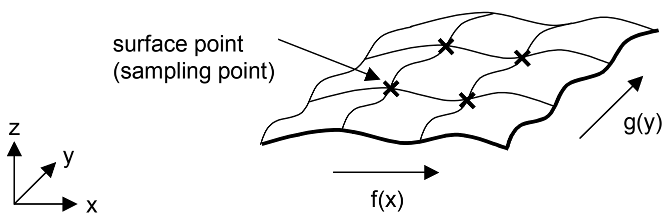
It is important that the number of base points is high enough to represent the waviness in a correct way. The generated nodes or mathematical functions are superposed for each contact surface. In this manner there exist two resulting functions for a mating contact, one for each contact surface. These two functions can be put together with a phase parameter for the assembly variants or to simulate the kinematical behaviour of the system. With the use of a commercial system for tolerance chain calculation VisVSA™ [N.N. 2003] it can be shown, in what way elastic deformations and tolerance chain calculation are coupled.

3.1 One-Dimensional Contact Situation

Previously only line-functions could be considered, so that the original conversion possesses validity only for the one-dimensional case. Such an equation must be set up for both the elastic deformations and for tolerance zones. Subsequently the equations can be superposed through superposition for each involved contact surface wherefrom an integral description of the involved mating surfaces results. The four statistical moments (mean value, standard deviation, skewness, kurtosis) that are necessary for the integration of the gathered information into a simulation system, are calculated mathematically out of the total equation. These can be integrated into a commercial tolerance analysis system that represents the kernel of the hybrid simulation [Lustig 2003].

3.2 Representation of Contact Surfaces with Point Coordinates

As already mentioned above, point data out of discretised surfaces can be transferred into a function equation. Previously this possessed validity to the one-dimensional-case, which is too far away from the description of real conditions. On this account a two-dimensional view of all involved contact surfaces must result from this circumstance. Since the original data to generate the above described line-function remains the same, there has to be made a success in connecting any amount of surface-coordinates (-points) in a suitable manner. Basically different mathematical tools can be used in this context. Some strategies are described as follows.
The base of the executed views is formed by the approach that surfaces can be represented through multiplication of two (lines-) functions, one for each coordinate direction in the right-angled coordinate system (figure 3) [Schloder 2003].



$$z = f(x)\, g(y) \Rightarrow F(x,y,z)$$

Figure 3. Surface functions out of two line functions

The generation of a surface function can be generated from the mathematical point of view through the use of interpolation and approximation, whereby the approximation does not consider the real position of a surface point, but only produces a function from which all surface points have a minimal distance. Therefore this procedure is excluded.
All interpolation procedures have common that always differently natured polynoms are multiplied in order to receive a total equation.
In the interpolation with polynoms of the order n, the order of the surface function arises through the amount of sampling points. Here certain function parameters must be determined, which can be calculated by inserting the coordinates of the sampling points and subsequently solving the resulting equation system.
The interpolation with Lagrange-polynoms requires other functions as basis functions. It is demanded that they have the value one at the sampling point and at all other nodes the value zero. These Lagrange-polynoms are also multiplied in the two-dimensional area in order to receive a surfaces function. In a similar manner, but only under use of other basis functions the interpolation with Newton-polynoms can be carried out.

In the practical use of these procedures there often appear distinct weaknesses regarding the demanded quality of the surface function, what has manifold causes. When using polynoms of the order n or Lagrange-polynoms, all polynomial elements must be newly calculated when the data of the sampling points changes, because each element depends on all sampling points. This requires a high effort in time and in calculation process. On this account the interpolation with the use of Newton-polynoms is mostly used, because when the number of sampling points is increased only the corresponding amount of elements have to be added.
Another problem lies in the number of sampling points itself. Generally the order of the polynom of the surface function generated by using the presented interpolation methods depends on the amount of the related surface points. Accordingly with a slight number of surface points a low polynomial order can be expected, with a large amount of points the polynomial order will be very high. Interpolation-functions tend to a stronger waviness with the increase of the polynomial order. With the increasing number of interpolation-points (that means increase of the polynomial order), the waviness considerably increases therefore. The results for surface functions determined by interpolation are therefore useless for further application, because the overshooting within one surface is simply to high. For this reason the determined surface functions generated through interpolation describe a real component surface too inexactly for further consideration.
Because of the fact that out of given point data no reasonable surface function for future application can be determined, the further proceeding will deal with the direct superposition of the point coordinates. The point data determined out of the elastic deformations and the fabrication contingent surface-deviations are coupled additive individually and result therefore in a point data model of a real component surface. The statistical moments can be calculated directly out of this data and can be integrated into a tolerance analysis system for further processing and simulation.

4. Method on the example

The example was presented in figure 1. It is a simplified representation of a technical system, e.g. for turning or for other cutting processes. The main drive for the workpiece is placed on the left side, the tooling support is placed on the right side.

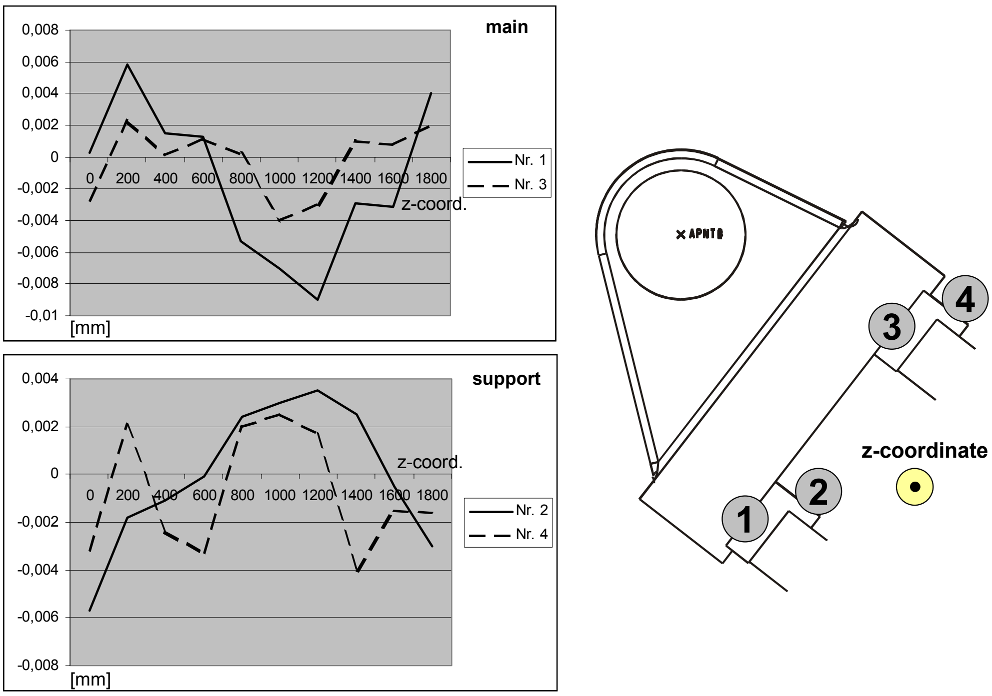


Figure 4. Motivation on the example: Model with tolerance based deviations

The cutting process is modelled with a cutting force of 150 kN and a feed force of 60 kN. The forces are coupled into the structure by bar-elements. The aim is the evaluation of reaction forces in the guidelines by the simulation model. The number of supporting points to get deviations can vary from case to case. And of course these points have a correlation with the hardpoints for Finite-Element-Calculation. The reaction forces of the cutting process in the guidelines are calculated on this base.
Both guidelines (upper and lower) are influenced by tolerance deviations, as shown in figure 4, in two directions as a function of z-coordinate or given as a discrete table.
The superposition of both effects for the calculation model was generated as an summary of the deviation of the nodes. The generation of a surface function would be possible and effective for a two-dimensional problem. For the mentioned functional-oriented criteria on kinematical behaviour the elastic deviation and the tolerance based deviation were shifted relative to each other by 200mm. The calculation was performed on VisVSA™.

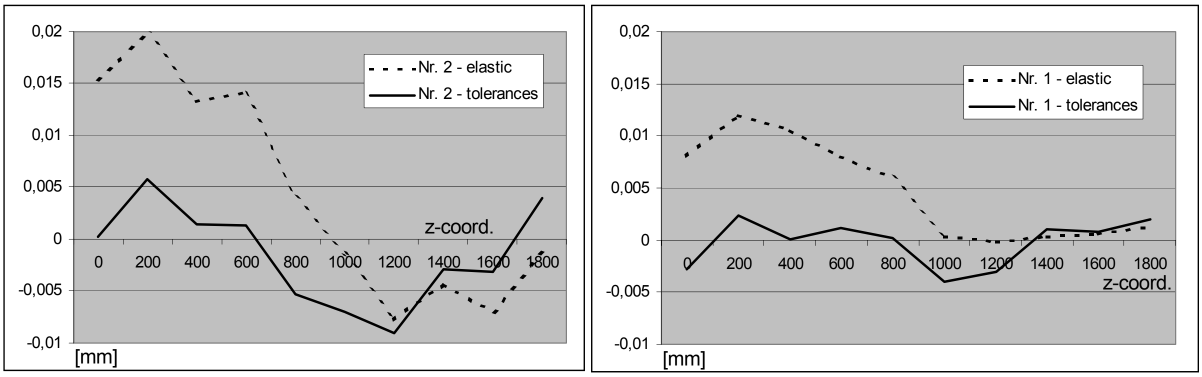


Figure 5. Results for the Example

It is obvious that the effect is not negligible and the cutting result is directly dependent on the mentioned factors. The deviations vary in a field of 30µm for the first guideline and 15µm for the second guideline after the superposition (figure 5 and 6); whereas the elastic tolerance based deviations vary in a field of 15µm and 8µm, which is nearly half of the coupled deviation. By using these information the quality relevant deviation of concentricity gets a value of 21µm under both influences. The kinematical superposition of the effects results in a variation of the concentricity of both axis from 16µm to 21µm depending on the tolerance zones, elastic deformation resulting from forces and the kinematical status of the technical system.

offset shift	point	mean	deviation concentricity
0	1	0,064	**0,0021**
	2	0,064	
	3	0,064	
	4	0,0661	
200	1	0,0642	**0,0018**
	2	0,0644	
	3	0,0639	
	4	0,066	
400	1	0,0641	**0,0016**
	2	0,0643	
	3	0,0636	
	4	0,0657	

[mm]

Figure 6. Concentricity as function of the offset shift

Further on the interpretation of results can be obtained along nodding, yaw, rolling to get an exacter behaviour in all three dimensions. The aim of this paper was to show the coupling of elastic deviations and tolerance based zones as well as the kinematical influence on an example.

5. Conclusion

The described method is suitable for the common simulation of elastic deformations and tolerance zones within functional relevant areas of a product. These influences were observed and optimized previously only separate and independent from one another. With this new acquisition, a combined sensitivity analysis and optimization becomes possible. How indicated, also further evaluation algorithms can be used case-specific. Based on the simple example of a strongly simplified tool machine it could be shown how both effects are superposed.

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Dipl.-Ing. Ralph Lustig
University Erlangen-Nuremberg, Institute for Engineering Design
Martensstraße 9, D-91058 Erlangen, Germany
Telephone: +49-(0)9131-8527987, Telefax: +49-(0)9131-8527988
E-mail: lustig@mfk.uni-erlangen.de