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STATISTICAL TOLERANCE ANALYSIS AND RESULT VISUALISATION FOR SYSTEMS IN MOTION

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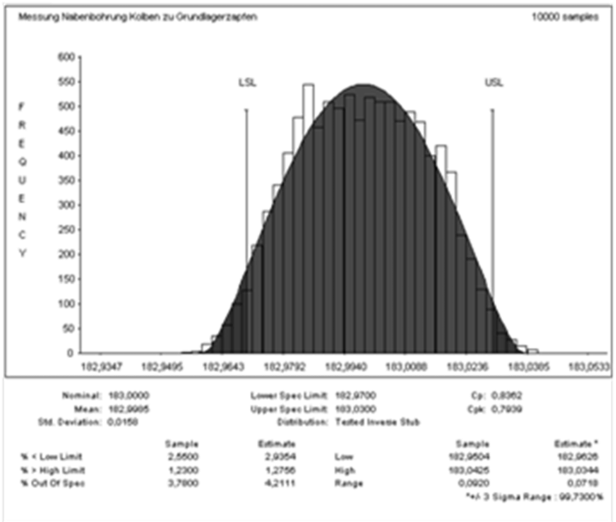
1. Introduction

During product development, the designer is "hunted" by a huge amount of requirements concerning product properties and process aspects. The dominating requirement is that the product has to fulfil a defined function at operating state. The focus in this paper is on mechanisms which are characterised by precisely coordinated motions of the components. Small geometrical deviations which originate from manufacturing discrepancies and from deformations of the components can degrade the motion accuracy and, consequently, can increase the danger of collisions. Against this background, the goal of tolerance analysis is on the one hand to ensure the functionality of the system and on the other hand to reveal potentials for an optimized tolerance allocation.
Nowadays, the usual situation for tolerance analysis approaches is that only a static assembly is considered and not a whole motion cycle of a mechanism. In the latter case, functional dimensions and contributors vary over the motion cycle. It is important to enlarge analysis procedures and result representation schemes to take this dynamic aspect into account. In addition to this, systems in motion are affected by acting forces which lead to deformations of the components and to defined relative displacements of the components at joints with clearance. An integrated model has to be used for dynamic tolerance analysis to get to more realistic results.
In this paper, a MATLAB®-tool for the statistical tolerance analysis of systems in motion in combination with an appropriate result visualisation shall be presented. The demonstrator is a crank mechanism of a combustion engine. Beside manufacturing deviations, elastic deformations and joint clearances are integrated in the tolerance analysis model. After a literature survey, the integrated tolerance analysis model and the analysis procedures are described in chapter 3. The following chapter then presents the implementation of the analysis and result representation procedures in MATLAB®. The last chapter summarizes the paper and points out future prospects.

2. Literature survey

Tolerance analysis can be divided into three basic steps. At first, the mathematical relation between the functional dimensions of the system and the different geometrical deviations has to be established (functional relationship or tolerance chain). It is important to realize that especially common geometrical tolerances are not suitable for mathematical calculations. This leads to the need of an explicit tolerance representation and a suitable transformation procedure. Existing representation schemes are shown in [Hong, et al., 2002] and [Prisco, et al., 2002].
The second step analyses the variations of the functional dimensions and the percent contribution of each contributor which affects the functional dimensions. For this purpose, different tolerance analysis

methods can be applied to the functional relationship [Shah, et al., 2007]. Statistical methods like Monte Carlo simulation, Linearized stack-up approach, Method of System Moments or Numerical Integration technique facilitate the integration of production information about the geometrical characteristics [Nigam, et al., 1995].
The last step is the representation and interpretation of the simulation results. In Computer-Aided Tolerancing (CAT) - systems, the results for the functional dimensions (histogram, information about mean value, scrap rate, process capabilities etc.) and the percent contributions are displayed as shown in Figure 1.



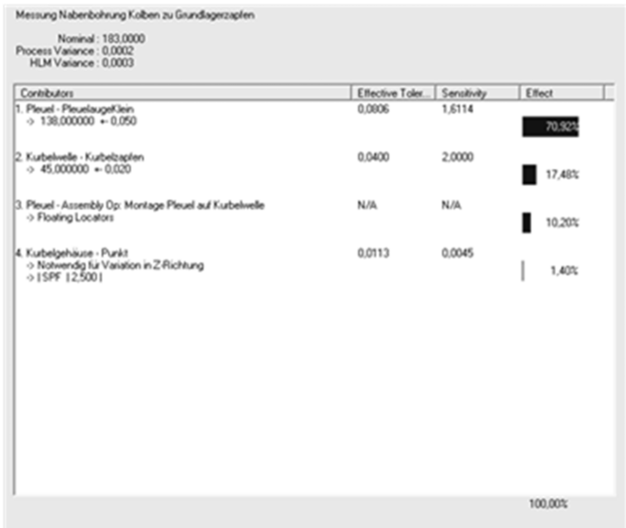


Figure 1. Common result representation in CAT-systems (in this case VisVSA®)

In the following, the focus is on geometrically non-ideal systems in motion which are affected by different kinds of deviation. The basic compatibility of multi-body systems and tolerance analysis methods is shown in [Hörsken et al. 1999]. In addition to manufacturing deviations, the motion behaviour of a mechanism with lower kinematic pairs can be influenced by joint clearances. A clearance vector is inserted into the functional relationship to describe the motion-depending relative displacement of the components which are connected by a joint with clearance. Its length and direction can be calculated by different joint models. For example, in [Choi et al. 1998] the clearance vector is calculated according to the acting forces in a lubricated revolute joint. Beside joint clearances, the functional dimensions are influenced by motion-depending deformations. This aspect is treated for

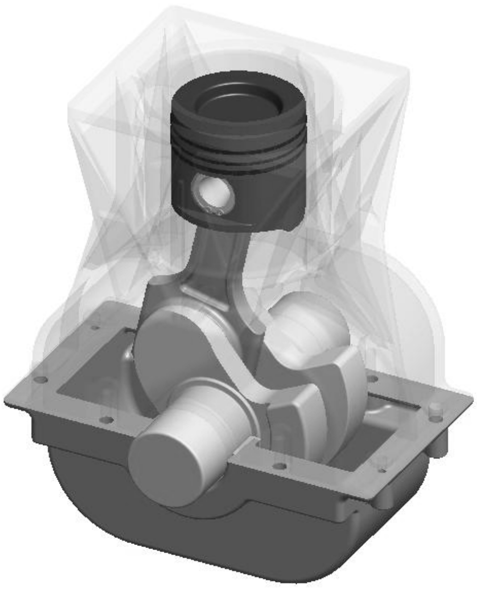
example in [Imani et al. 2009]. Mechanisms which are comprised of higher kinematic pairs in general and especially gears are treated in [Bruyère, et al., 2007] and [Sacks, et al., 1999].
It can be observed that all these activities deal with different aspects concerning the setting up of the functional relationship, the use of different tolerance analysis methods and the representation of the results. A holistic approach which integrates different modelling strategies for mechanisms with different kinds of deviation and the possibility to apply different tolerance analysis and result representation methods does not exist. An integrated tolerance analysis model for a crank mechanism considering manufacturing deviations, joint clearance and elastic deformation was set up in [Stuppy, et al., 2009]. This approach is the basis for the research work which will be presented in the following. The focus is on applying statistical tolerance analysis und result representation methods on this example.

3. Integrated tolerance analysis approach

For an integrated tolerance analysis of systems in motion, it is important to enlarge existing procedures for setting up the functional relationship.
In the following, the demonstrator used in this paper is briefly described. After that, the functional relationship for the functional dimension is set up taking into account different kinds of deviation. The chapter ends with the explanation of the analysis procedures.

3.1 Demonstrator crank mechanism

The demonstrator is a crank mechanism inside a single cylinder four-stroke combustion engine (see figure 2). The motion of the components is precisely coordinated. One motion cycle means that the crank mechanism passes all four strokes, the crankshaft performs two rotations and the piston moves twice from top dead centre to bottom dead centre and back. At φ = 360° the charge changing takes place.



Calculation parameters	Unit	Value
Crank radius r	mm	45
Conrod length l	mm	138
Piston diameter D_{pi}	mm	84
Piston mass m_{pi}	g	875
Conrod mass m_{cr}	g	805
Maximum cylinder pressure p_{max}	bar	180
Rotational speed n	min^{-1}	3000

Figure 2. Demonstrator crank mechanism: CAD-model and calculation parameters

Geometrical deviations of the components can lead to undesirable collisions. This aspect justifies the need of tolerance analysis. For the demonstrator crank mechanism, collisions of the piston and the valves or the piston and the balance weights of the crankshaft can occur. The problem is that the collision position of the piston is not easy to pre-estimate. For example, the collision point of piston and balance weights is not necessarily the bottom dead centre. Consequently, it is important to analyse among others the piston position in relation to the principal axis of the crankshaft as the functional dimension over the whole motion cycle of the mechanism.

3.2 Integration of different kinds of deviation in the functional relationship

For the piston position of the crank mechanism, the functional relationship shall be established taking into account the following deviations:

- Link length deviations of crank radius r = 45±0,02 mm (normal distribution) and conrod length l = 138±0,05 mm (trapezoidal distribution)
- Position deviation of cylinder axis e = ±0,02mm (triangular distribution)
- Joint clearance at conrod big end bearing s = 0,06±0,015 mm (uniform distribution)
- Elastic deformation of the crankshaft w

As shown in [Stuppy, et al., 2009], the kinematical relations, the relative displacement at the joint with clearance and the elastic deformation are represented by motion-depending vectors. The functional dimension is then the result of a stack-up of the several vectors according to the interaction of the components.

In Figure 3, the integration approach is shown. For the calculation of the clearance vector at the conrod big end bearing and the elastic deformation of the crankshaft, it is important to have knowledge of the acting forces in the mechanism. Especially, the joint force F_{pin} which acts from the conrod onto the crankshaft plays an important role. The clearance vector s_{HD} which describes the displacement of the conrod relative to the crankpin is calculated by the method of superposed hydrodynamic load portions of HOLLAND. For the calculation of the deflection w of the crankshaft, an analytical model is used.

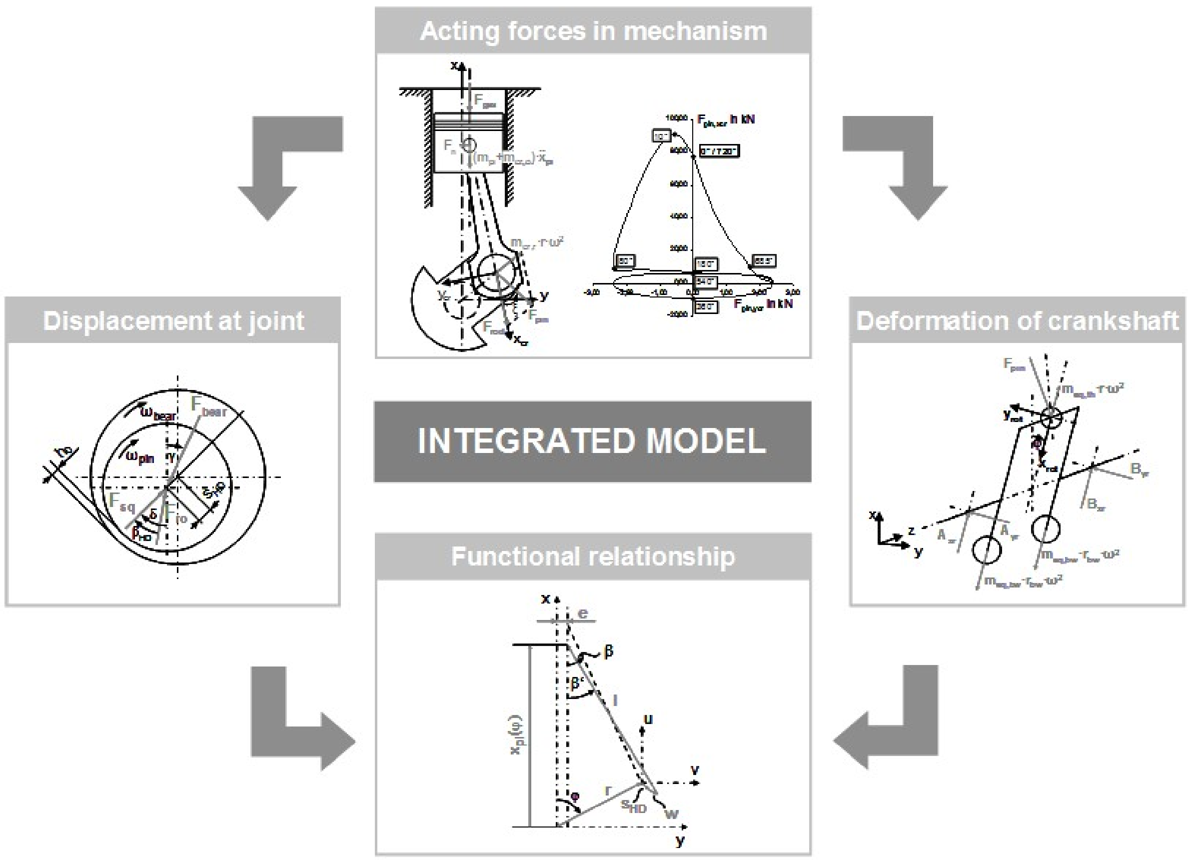


Figure 3. Procedure for integration of different kinds of deviation in functional relationship

The equation of the deviated piston position then is:

$$\begin{bmatrix} x_{pi}(\varphi) \\ y_{pi}(\varphi) \end{bmatrix} = \begin{bmatrix} r \cdot cos\,\varphi \\ r \cdot sin\,\varphi \end{bmatrix} + \begin{bmatrix} s_{HD,u} \\ s_{HD,v} \end{bmatrix} + \begin{bmatrix} w_u \\ w_v \end{bmatrix} + \begin{bmatrix} l \cdot cos\,\beta' \\ -l \cdot sin\,\beta' \end{bmatrix} \quad (1)$$

Due to the displacement at the conrod big end bearing and the deflection of the crankshaft in combination with the joint condition between piston and housing (piston moves on the deviated cylinder axis), the angle β changes a little bit. The modified angle β' can be determined by using the joint constraint $y_{pi}(\varphi) = e$.

3.3 Analysis procedure

The basic relationship between the deviated piston position and the different deviations was set up in the previous chapter. It is now important to define the global analysis procedure which means the calculation sequence for the several deviations before the background of applying tolerance analysis methods on the functional relationship afterwards.

The procedure and the tolerance analysis methods implemented as shown in chapter 4 are depicted in figure 4. Due to the fact that the manufacturing deviations of r, l and e are very small compared to the main dimensions of the mechanism, the acting forces are calculated based on the ideal kinematics. Depending on the forces, the displacement at the conrod big end bearing and the elastic deformation of the crankshaft are evaluated separately and inserted, together with the deviations of r, l and e, into the functional relationship as shown in the previous chapter.

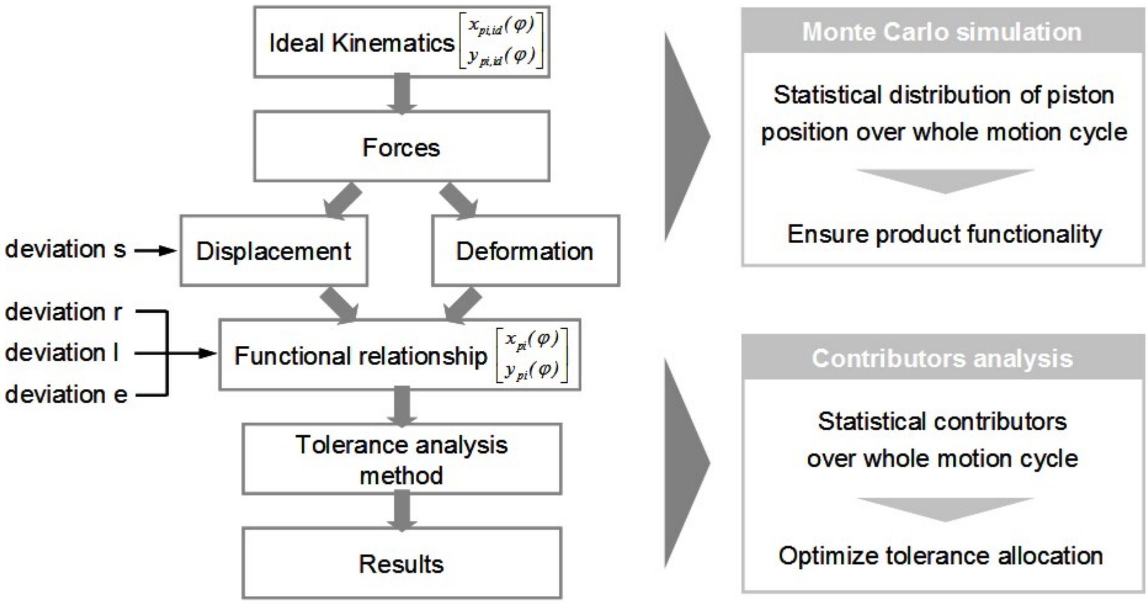


Figure 4. Tolerance analysis procedure and implemented methods

By applying tolerance analysis methods, two different results can be achieved:

1. Via the Monte Carlo simulation, the distribution of the functional dimension (piston position) over the whole motion cycle is evaluated. This result is important when the designer wants to ensure the product functionality.
2. By using contributors analysis methods, the statistical contributions of each toleranced parameter over the whole motion cycle are achieved. This result is the basis for an optimization of tolerance allocation.

In the next chapter, the implementation of these statistical tolerance analysis methods and an appropriate result representation in MATLAB® is shown.

4. MATLAB®-tool for statistical evaluation of functional dimensions and contributors

Instead of one assembly situation, for systems in motion a huge amount of mechanism positions have to be considered. For the crank mechanism, the distribution of the piston position and the contributors then have a dynamic character which means that they vary over the motion cycle. This aspect is absolutely important for the implementation of tolerance analysis and result representation methods as will be shown in the next sections.

4.1 Monte Carlo simulation

The Monte Carlo simulation is a widely-used method for the statistical analysis of the functional dimensions. With this method any input distribution of the toleranced parameters and linear as well as nonlinear functional relationships can be handled. Random values for the toleranced parameters are generated according to the distribution of each parameter. Each set of random values of the toleranced parameters is then used to calculate the functional dimensions via the functional relationships.

Figure 5 shows the procedure of the implemented Monte Carlo simulation for the analysis of the piston position. The implementation is characterised by a matrix-based approach which is more efficient concerning the calculation time. Matrix-based approach means that all the scalar equations for the determination of the piston position are transformed into a matrix-based formulation so that calculation loops are avoided.

When starting the program *montecarlo()*, the number j of geometry variations which are generated/analysed and the crank angle range phi which is considered have to be defined. At first, the pressure curve and the input parameters (e. g. mechanism dimensions, distributions of the toleranced parameters, rotational speed etc.) are imported and stored in the field variable *input*. After that, four different random number generators produce j values for each toleranced parameter r, l, e and s according to the manufacturing distributions. The four jx1 vectors are also stored in a field variable (*geom_var*). The values of the same row in the different jx1 vectors characterise one deviated crank mechanism. All in all, j different mechanism configurations are analysed afterwards. For each set of random values, the deviated piston position over the whole motion cycle is determined by the functional relationship as shown in chapter 3. The result is the jx721 matrix *x_pi* with j piston positions for each crank angle in the range from 0°-720° (1°-steps). Beside the deviated values of the piston position, the ideal and the nominal piston position over the motion cycle (1x721 vectors) are calculated because they are needed for further result evaluation and representation. The difference between them is that the deformation and joint clearance are taken into account when the nominal piston position is calculated whereas only the ideal kinematics (without clearance and deformation) are considered when the ideal piston position is determined. The output of each step is stored in the *result.mat* file.

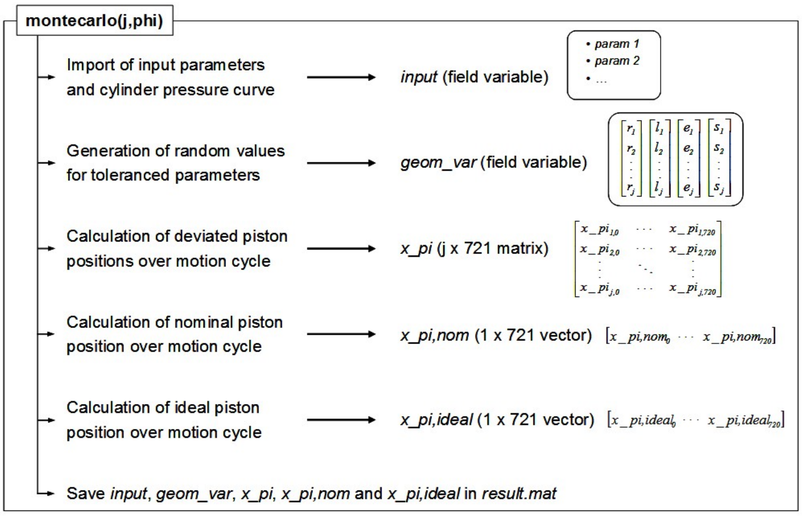


Figure 5. Procedure for Monte Carlo simulation

It is important that the number j of simulated geometry configurations is large enough to get reliable results (order of magnitude: 10.000). It is then possible to determine specific values for each crank angle from the matrix *x_pi*, for example mean values, standard deviations, scrap rates etc.

4.2 Contributors analysis

Beside the information about the distribution of the functional dimension, the contribution of each toleranced parameter to the variation of the functional dimension is an important aspect. For this purpose, contributors analyses are performed. The statistical contribution B_i of a toleranced parameter can be calculated according to the following equation:

$$B_i = \frac{\alpha_i^2 \cdot Var(M_i)}{\sum_{i=1}^{m} \alpha_i^2 \cdot Var(M_i)} \cdot 100\% \tag{2}$$

B_i: contribution of toleranced parameter M_i
M_i: toleranced parameter
m: number of toleranced parameters
α_i: linearity coefficient, that means partial derivative of functional relationship with respect to concerning toleranced parameter
$Var(M_i)$: variance of toleranced parameter M_i

For the crank mechanism, the contributions of r, l, e and s are evaluated according to this equation. The linearity coefficients change over the motion cycle and lead to motion-depending contributions of the toleranced parameters. The results (1x721 vectors for the contribution of each toleranced parameter over the motion cycle) are also stored in the file *result.mat*. This file is then the input file for the result representation tool which will be described in the next chapter.

4.3 Evaluation and visualisation of results

In chapter 2, common result representations for the distribution of the functional dimension and the contributors in CAT-systems were shown. In mechanisms, these characteristics change over the motion cycle. Consequently, it is not enough to visualise only one assembly situation.
A first approach to achieve an integrated visualisation of statistical and dynamic information of the functional dimension and the contributors is displayed in figure 6. This Graphical User Interface was set up in MATLAB® for the evaluation of the results gained by the analyses described before.

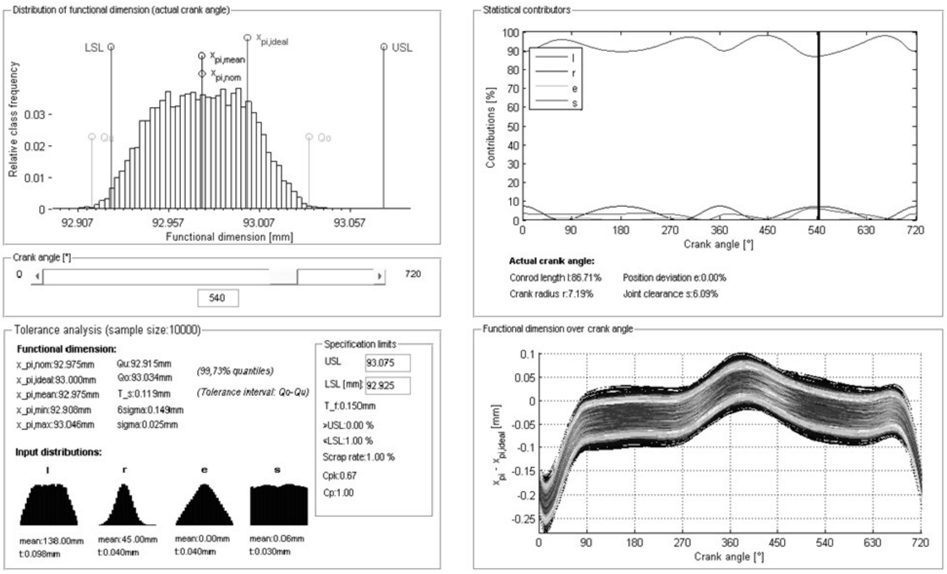


Figure 6. Graphical User Interface for dynamic tolerance analysis result representation

The GUI is subdivided into five different sections:

- Histogram of functional dimension at considered mechanism position (upper left)
- Slider for selection of mechanism position which shall be considered
- Specific values of functional dimension at considered mechanism position (also in relation to the crank angle depending specification limits) and input distributions (bottom left)
- Statistical contributors over motion cycle (upper right)
- Distribution of functional dimension over motion cycle (bottom right)

In the following, the different sections are briefly described:

Histogram

The histogram gives a quick overview of the piston position distribution at the currently considered mechanism position. Beside the distribution shape, the mean value, nominal value, ideal value and the 99,73% quantiles as well as the specification limits are shown.

Specific values

The specific values displayed in the bottom left corner provide a deeper insight for the actual mechanism position. All the values associated with the functional dimension and the specification limits vary over the motion cycle.
The nominal and ideal piston positions are loaded from the input file *result.mat*. The mean, maximum and minimum values are calculated from the deviated piston positions by using the MATLAB®-functions *mean()*, *min()* and *max()*. The standard deviation sigma is calculated by the MATLAB®-function *std()*. The limits Q_u and Q_o are determined by an implemented function in such a way that 99,73% of the simulated piston positions are located in-between them. Assuming an acceptance probability of p=0,9973 for the distribution of the functional dimension, the statistical tolerance T_s then is the difference Q_o-Q_u.
For normally distributed functional dimensions, the range of 6sigma plays a special role: 99,73% of all values are within a 6sigma-interval which is symmetric relative to the expected value. Consequently, a check of the distribution shape of the piston position can be carried out by comparing the values T_s and 6sigma. If the values T_s and 6sigma are equal, the functional dimension is normally distributed.
The region "specification limits" on the right hand side of the specific values can be used for the evaluation of the simulated piston positions with regard to the specification limits. For each crank angle, the values USL (upper specification limit) and LSL (lower specification limit) can be defined by the user. The functional tolerance T_f is the difference USL-LSL. An implemented function then counts the number of simulated mechanisms for which the deviated piston position is larger than USL (>USL) and lower than LSL (<LSL) respectively. The overall scrap rate is the sum of both. The process capability indices c_p and c_{pk} measure the location and extension of the distribution of the piston position related to the specification limits.

Contributors

The diagram on the upper right shows the motion depending contributions of r, l, e and s. Additionally, the exact values of the contributions for the actual crank angle are displayed below the diagram. The actual crank angle is marked by a vertical black line in the diagram.

Functional dimension over crank angle

The last section on the bottom right shows the difference between deviated and ideal piston positions over the whole motion cycle. The different colours give information about the probability of the several deviated piston positions for each crank angle. The diagram can be rotated by the user to get a 3-dimensional impression of the piston position distribution over the whole motion cycle of the crank mechanism (see figure 7).
By switching the modus in the source code, the diagram on the bottom right can be exchanged by another diagram which is depicted in figure 8. This diagram shows the courses of some specific values for one motion cycle. They visualise the location of the deviated piston positions relative to the

specification limits. Due to the fact that the deviations are very small, the values are scaled by a factor of 300. The vertical black line represents, like in the diagram of the contributors, the considered crank angle.

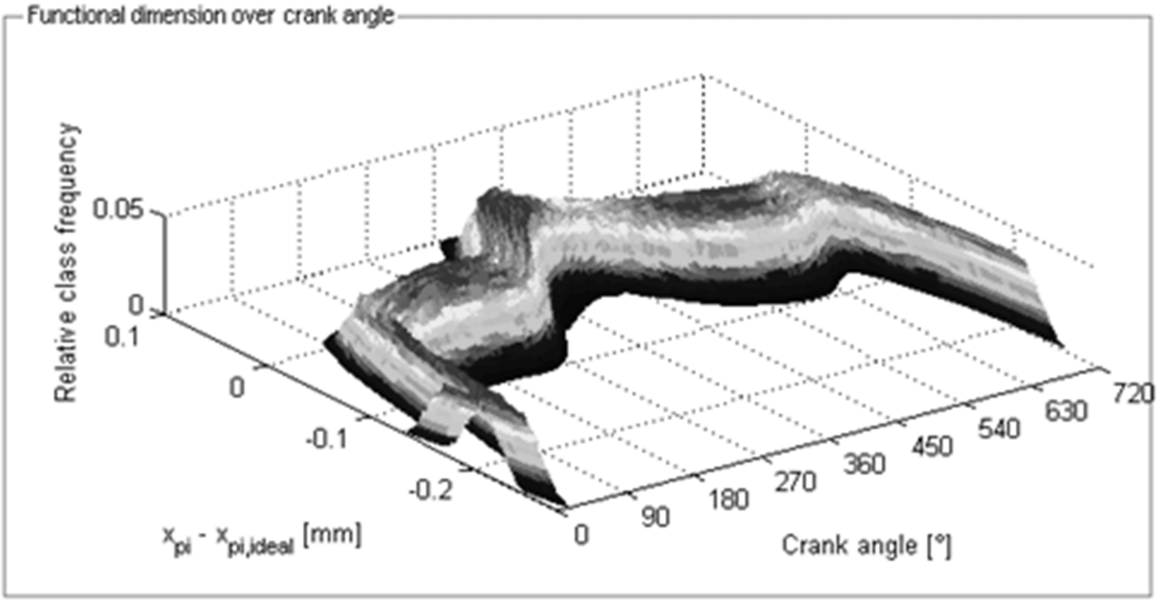


Figure 7. Difference between deviated and ideal piston positions (3D-view)

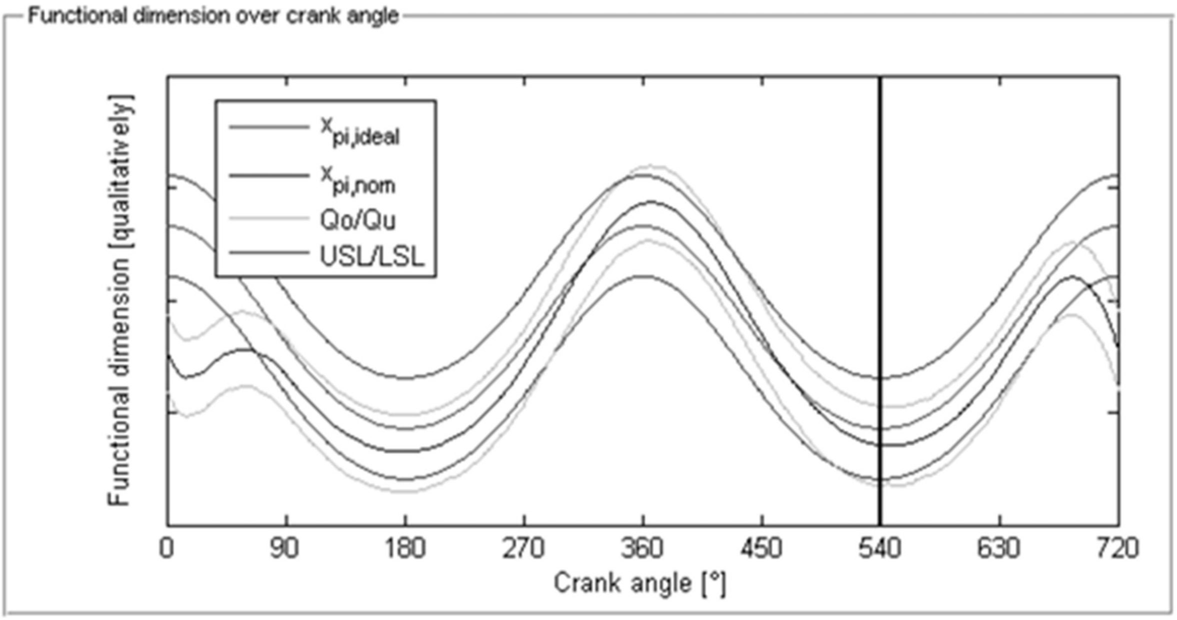


Figure 8. Specific values over crank angle

By analysing mechanisms and visualising the gained results in the described manner, it is possible to achieve a motion depending, statistical evaluation of the motion accuracy which can be used for ensuring the functionality of the system and improving tolerance allocation.

5. Summary and future prospects

In this paper, an integrated tolerance analysis for a crank mechanism in combination with an appropriated result visualisation set up in MATLAB® were presented. The piston position was analysed statistically depending on manufacturing deviations, joint clearance at the conrod big end

bearing and elastic deformation of the crankshaft. The integrated tolerance analysis model and the analysis procedure were presented in chapter 3. Based on this, chapter 4 showed the implementation of the Monte Carlo simulation and the statistical contributors analysis. For the evaluation and visualisation of the gained results a Graphical User Interface was implemented.
In the next steps, approved simulation tools shall be coupled to get to an integrated, computer aided tolerance analysis process for technical systems in motion. To maximize the benefit of tolerance simulation, it is important to optimally integrate the analysis procedures into the product development process. Therefore, the synchronisation of tolerance analysis and product development activities shall be investigated.

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